Physics 137B: The Structure of the Proton

- Studing the Proton Structure
- Electromagnetic Form Factor
- Elastice ep scattering
- Deep Inelastic Scattering

The Proton is Not a Point-like Particle

- Quark model says p consists of 3 quarks (but are they real?)
- Gyromagnetic moment $g_p = 5.586$ is far from the Dirac value of 2 that holds for pointlike spin-$\frac{1}{2}$ particles
- Size of nucleus consistent with nucleons of size ~ 0.8 fm

To study structure of the proton, will use scattering techniques

Similar idea to Rutherford's initial discover of the nucleus

Scattering of Pointlike Particles

- Rutherford Scattering (spinless electron scattering from a static point charge) in lab frame:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4(\frac{1}{2}\theta)}$$

where E is the enrgy of the incident electron and θ is the scattering angle in the lab frame

- Mott Scattering: Taking into account statistics of identical spinless particles

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\frac{1}{2}\theta)}{4E^2 \sin^4(\frac{1}{2}\theta)}$$

- Elastic Scattering of a spin-$\frac{1}{2}$ electron from a pointlike spin-$\frac{1}{2}$ particle of mass M:
  - Scattering of electron from static charge changes angle but not energy
  - For target of finite mass M, final electron energy is

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2(\frac{1}{2}\theta)}$$

  and the four-momentum transfer is

$$q^2 = -4EE' \sin^2(\frac{1}{2}\theta)$$

The elastic scattering cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\frac{1}{2}\theta)}{4E^2 \sin^4(\frac{1}{2}\theta)} \frac{E'}{E} \left[1 - \frac{q^2}{2M^2}\tan^2(\frac{1}{2}\theta)\right]$$

What Happens if the Target Particles Have Finite Size?

- Suppose the charge distribution is $\rho(r)$ normalized so that $\int \rho(r) d^3r = 1$
- The scattering amplitude is modified by a "Form Factor"

$$F(q^2) = \int d^3r e^{i\vec{q}\cdot\vec{r}} \rho(r)$$

  So that the cross section is modified by a factor of $|F(q^2)|^2$

- Note: As $q^2 \to 0$, $F(q^2) \to 1$
- We therefore can Taylor expand

$$F(q^2) = \int d^3r \left(1 + i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + \ldots\right) \rho(r)$$

- The first $\vec{q}\cdot\vec{r}$ term vanishes when we integrate

$$\begin{aligned}
F(q^2) &= 1 - \frac{1}{2}\int r^2 dr d\cos\theta d\phi \rho(r)(qr)^2 \cos^2\theta \\
&= \frac{2\pi}{2}\int dr d\cos\theta q^2 r^4 \cos^2\theta \\
&= 1 - \frac{< r^2 >}{4} q^2 \int \cos^2\theta d\cos\theta \\
&= 1 - \frac{< r^2 >}{4} q^2 \left[\frac{\cos^3\theta}{3}\right]_{-1}^{1} \\
&= 1 - \frac{< r^2 >}{6} q^2
\end{aligned}$$

  This F is called the "form factor"

- Thus, if we plot $\frac{d\sigma}{d\Omega} / \frac{d\sigma}{d\Omega}_{pointlike}$ vs $\tan^2 \frac{1}{2}\theta$ or vs q^2 we can measure the size of the proton

$$< r^2 >^{\frac{1}{2}} = 0.74 \pm 0.24 \times 10^{13} \text{ cm} \sim 0.7 \text{ fm (McAllister and Hofstadter, 1956)}$$

  See the next two pages for relevant plots

Hoffstader and McAllister's Experimental Setup

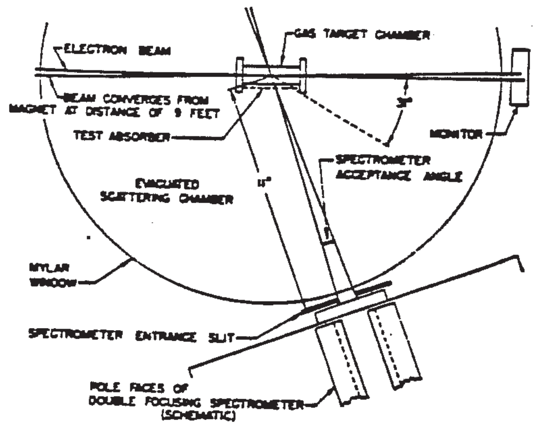


FIG. 2. Arrangement of parts in experiments on electron scattering from a gas target.

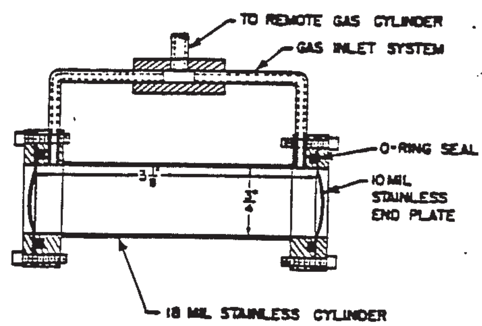


FIG. 1. Basic design of the gas chamber.

Hoffstader and McAllister's Results

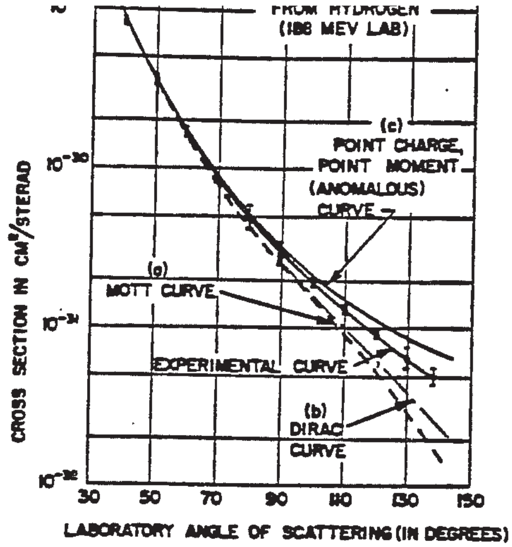


FIG. 5. Curve (a) shows the theoretical Mott curve for a spinless point proton. Curve (b) shows the theoretical curve for a point proton with the Dirac magnetic moment, curve (c) the theoretical curve for a point proton having the anomalous contribution in addition to the Dirac value of magnetic moment. The theoretical curves (b) and (c) are due to Rosenbluth.[8] The experimental curve falls between curves (b) and (c). This deviation from the theoretical curves represents the effect of a form factor for the proton and indicates structure within the proton, or alternatively, a breakdown of the Coulomb law. The best fit indicates a size of 0.70×10^{-13} cm.

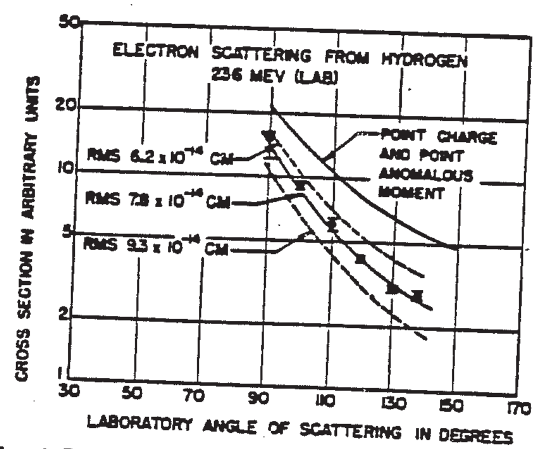


FIG. 6. This figure shows the experimental points at 236 Mev
d the attempts to fit the shape of the experimental curve. The
t fit lies near 0.78×10^{-13} cm.

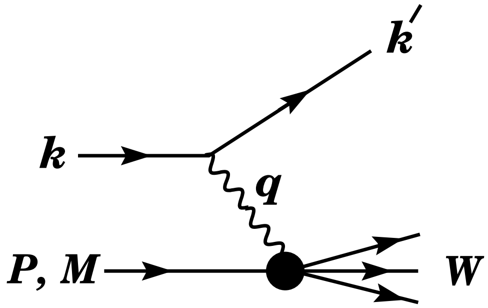
Angular Distributions

- In addition to total x-section, can look at angular dependence
  - For elastic scattering, the angle uniquely determines the energy of the outgoing electron
  - So angle is the only independent variable
- Can write down the most general form of the cross section

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2(\frac{1}{2}\theta)}{4E^2 \sin^4(\frac{1}{2}\theta)} \frac{E'}{E} \left[W_2(Q^2) + 2W_1(Q^2) \tan^2(\frac{1}{2}\theta) \right]$$

- These W are called the proton form factors
- Understanding these two form factors tells us about the structure of the proton!

Inelastic Scattering



- Once the proton breaks up, the energy of the outgoing electron is not determined just from the angle of the scattering
- We have an additional degree of freedom: the invariant mass of the hadronic system
- In lab frame: proton 4-momentum is $P = (M, 0)$
- In any frame, four-momentum transfer is $k = k' + q$ and the four-momentum of the final hadronic system is $W = p + q$
- Invariants of the problem:

$$\begin{aligned} Q^2 &= -q^2 = -(k-k')^2 = 2EE'(1-\cos\theta) \\ P\cdot q &= P\cdot(k-k') = M(E-E') \end{aligned}$$

  where the the last expression in each row is evaluated in the lab frame.

- Define $\nu \equiv E - E'$ so $P \cdot q = m\nu$ and

$$\begin{aligned} W^2 &= (P+q)^2 = (P-Q)^2 = M^2 + 2P\cdot q - Q^2 \\ &= M^2 + 2M\nu - Q^2 \end{aligned}$$

  where $Q^2 = -q^2$

- Elastic scattering corresponds to $W^2 = P^2 = M^2$ so for elastic scattering $Q^2 = 2M\nu$
- For inelastic scattering, we can define 2 indep dimensionless parameters

$$
\begin{aligned}
x &\equiv Q^2/2M\nu \\
y &\equiv \frac{P \cdot q}{P \cdot k} = 1 - E'/E
\end{aligned}
$$

Structure Functions

- We can express the x-section for DIS

$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2}\frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}\left[W_2(q^2,W) + 2W_1(q^2,W)\tan^2(\frac{1}{2}\theta)\right]$$

- These are the same two terms as for the elastic scattering, except that the W now depend on both q^2 and W.
- W_1 and W_2 care called the *structure functions*
- A SLAC-MIT group measured $d\sigma/dq^2 d\nu$ at 2 angles: 6° and 10° (see next page for the plots)
  - Surprise: Above the resonance region, σ did <u>not</u> fall with Q^2 !
  - Like Rutheford scattering, this is evidence for hard structure within the proton
  - To deternine W_1 and W_2 separately, would need to measure at 2 values of E' and of θ that give the same q^2 and ν
  - The first exp couldn't do this: small angle where experiment ran, W_2 dominates so study that
  - Most important result: W_2 depends only on the dimensionless combination $x = Q^2/2M\nu$ (or $\omega = 2M\nu/Q^2$) "scaling"

See the next 2 pages for the experimental results

SLAC-MIT Results: W Distribution for Different q^2

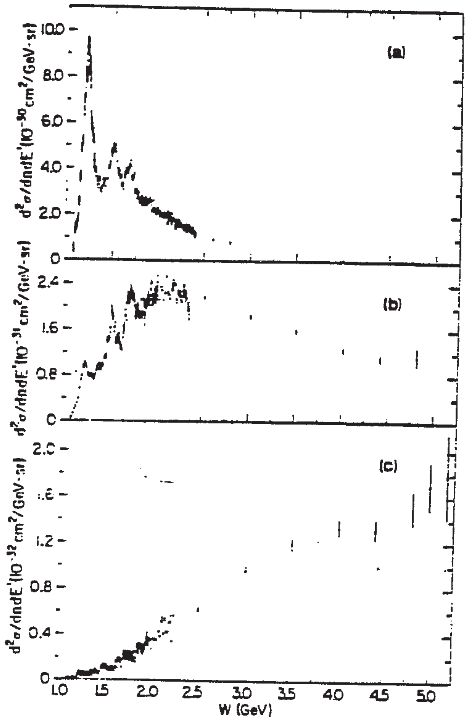
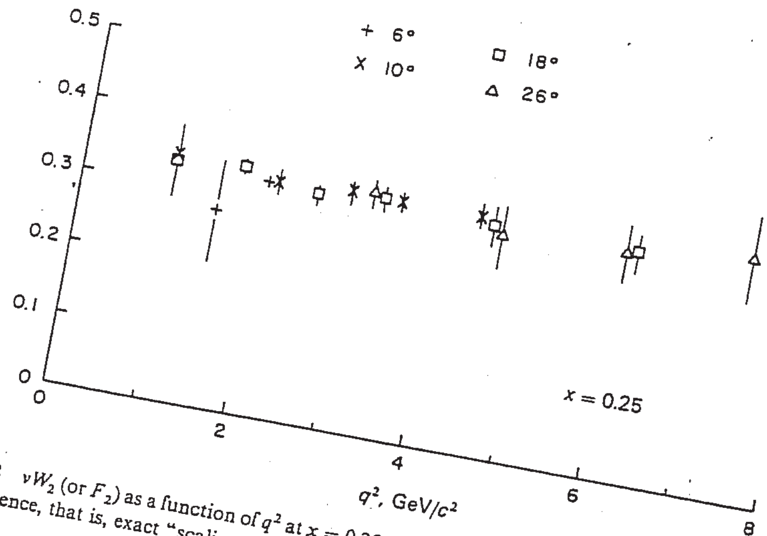


FIG. 2. Three representative radiatively corrected spectra at (a) $\theta = 6°$, $E = 7$ GeV; (b) $\theta = 6°$, $E = 16$ GeV, and (c) $\theta = 10°$, $E = 17.7$ GeV. The ranges of q^2 covered are (a) $0.2 \leq q^2 \leq 0.5$ (GeV/c)^2; (b) $0.7 \leq q^2 \leq 2.6$ (GeV/c)^2; and (c) $1.6 \leq q^2 \leq 7.3$ (GeV/c)^2. The elastic peaks are not shown.

SLAC-MIT Results: Scaling



'e 8.8 νW_2 (or F_2) as a function of q^2 at $x = 0.25$. For this choice of x, there is practically no pendence, that is, exact "scaling." (After Friedman and Kendall (1972).)

Evidence for scaling:

$\frac{F_2(x)}{x}$ depends on x but not q^2

$\Rightarrow$ "Hard" pointlike partons inside nucleon

What does Scaling Tell Us?

- Supposed there are pointlike partons inside the nucleon
- Let's work in an "infinite momentum" frame so we can ignore all mass effects
- In the infinite momentum frame, the roton 4-momentum: $P = (P,0,0,P)$
- Visualize a stream of parallel partons each with 4-momentum xP where $0 < x < 1$; neglect transverse motions of the partons
  - x is the fraction of the proton's momentum that the parton carries
- Suppose our electron elastically scatters from a parton

$$
\begin{aligned}
(xP+q)^2 = -m^2 &\sim 0 \\
x^2P^2 + 2xP\cdot q + q^2 &= 0
\end{aligned}
$$

Since $P^2 = M^2$, if $x^2M^2 << q^2$ then

$$
\begin{aligned}
2xP\cdot q &= -q^2 = Q^2 \\
x &= \frac{Q^2}{2P\cdot q} = \frac{q^2}{2M\nu}
\end{aligned}
$$

This x is the same x we defined before

Scaling of the Structure Functions is evidence for the presence of pointlike partons with the proton!

Some comments:

- We are using an impulse approximation where the scattering occurs before the partons have a chance to redistribute themselves
- We implicitly assume that after the scattering, the partons that participate in the scattering turn into hadrons with probability=1
- This is a lowest order calculation. We will see later that to higher order in perturbatin theory, QCD corrections will introduce slow scaling violations.

Some Facts About Parton Distribution Functions

- Let $f(x)$ be the prob of finding a parton with mom fraction between x and $x+dx$ in the proton. Then because the partons together carry all the momentum of the proton

$$\int dx\, x f(x) = \int dx\, x \sum_i f_i(x) = 1$$

  where $\sum_i$ is a sum over *all* partons in the proton

- We call $f(x)$ the parton distribution function since it tells us the momentum distribution of the parton within the proton

- It's natural to associate the partons with quarks, but that's not the whole story

  – Because ep scattering occurs through the electromagnetic interaction, it only occurs via scattering with charged partons. If the proton also contains neutral partons, the EM scattering won't "see" them

  – Let's assume that the ep scattering occurs through the scattering of the e off a quark or antiquark

    ∗ We saw that the SU(3) description of the proton consists of $2\ u$ and $1\ d$ quark.

    ∗ However we can in addition have any number of $q\bar{q}$ pairs without changing the proton's quantum numbers

    ∗ The 3 quarks (uud) are called *valence quarks*. The additional $q\bar{q}$ pairs are called *sea* or *ocean* quarks.